since no automatic contrivance can allow for the errors caused by change of refraction and flexure, and absolutely uniform rotation of the polar axis is of less consequence. It is sufficient if the errors of the clock-work are small, and are not cumulative, as they are apt to be without control by a standard clock.

It is not easy to say how readily this control could be applied to smaller instruments. The governors of driving-clocks seldom rotate so slowly as once a second, but the control could evidently be applied to any shaft rotating in an integral part of a second. The sector might also require to be balanced by a counterpoise, or two diametrically opposite sectors and magnets could be used. For large telescopes, experience at this Observatory has shown that the control is perfectly efficient.

A method for bringing the control of the Lick telescope rapidly into action, and applicable under existing arrangements, occurred independently to Professor Holden and myself, and will be given a trial. Four equidistant sectors, each consisting of a block of soft iron, are enclosed between two circular discs of thin sheet brass, the axis of the governor passing through the center. The sector nearest to the magnet will then come first into action, the others being inoperative, and the governor will never have to gain more than one-fourth of a revolution; the chances are that a gain of less than one-fourth will be required.

DETERMINATION OF THE RELATION BETWEEN THE EXPOSURE-TIME AND THE CONSEQUENT BLACKENING OF A PHOTOGRAPHIC FILM.

By Armin O. Leuschner.

For some time past it has been the custom at the Lick Observatory to standardize photographic plates on which photographs of celestial objects had been secured. The process of standardizing a plate consists in exposing some portion of it, previously protected against the light of the object, to the light of a standard-lamp through a small square aperture. On development the plate will show a more or less dark square, due to the light of the lamp. In all cases a series of squares is secured on the plates, the times of exposure being either 1^s, 5^s, 10^s, 20^s, 30^s, etc., or 1^s, 2^s, 4^s, 8^s, 16^s, etc., according to cir-

cumstances. The various parts of the image of the object are then matched with the different squares, and thus the actinic intensity of the object can be found in terms of the intensity of the standard-lamp; the unit of light being the amount of light received through an aperture, one mm. in radius, one m. distant, in one second. If this unit has once been ascertained in terms of the intensity of any celestial object, we may refer all our results to the intensity of that object, as the Moon, or *Polaris*, for example.

It was after the Total Solar Eclipse of the year 1886, that Mr. W. H. PICKERING, of the Harvard College Observatory, for the first time exposed some eclipse-plates to a standard-lamp, with a view of determining the actinic intensity of the corona and the surrounding sky. The circumstances under which Mr. PICKERING was compelled to try his experiment were most unfavorable. Besides the eclipse-plates, another plate was standardized, in order to secure the true intensities for the various exposure-times, as it was feared that the eclipse-plates had suffered from exposure to a damp climate. In fact, Mr. PICKERING found that there *was* a difference in the intensities of the squares on the two plates, for he says:

"On comparing the standard squares with those upon the eclipse-plate, the latter were found to be considerably weaker, the denser squares showing the falling off more than the fainter ones. This may have been due in part to the original difference in the sensitiveness of the plates, but I think it may very likely have been caused by the heat and moisture to which the eclipse-plates had been exposed."

What I wish to dwell upon here is the assertion that the difference in the intensities of the squares *may* have been due to an original difference in the sensitiveness of the plates. In fact, we shall soon see that a difference in sensitiveness does not only exist in different plates of a certain species, but even in the various parts of the *same* plate.

To reduce the intensities of the various squares to any adopted unit, it is necessary to know the relation between the time of exposure and the consequent blackening of the photographic film,—the law according to which the intensities of a square increase with the exposure-times. It is generally assumed that the intensity is directly proportional to the exposure-time,—that is to say, a square exposed for four seconds will be twice as dark as one exposed for two seconds only. In a recent paper, presented before the B. A. A. S. meeting

of 1889, Capt. ABNEY reported that his experiments showed that intensity was proportional to the time, without limitations. Following the example of Mr. PICKERING, the Lick Observatory eclipse-plates at the Total Solar Eclipse of January 1st of this year were all standardized, and it was (on the authority of Capt. ABNEY) assumed that intensity varied as exposure-time, in the reductions of the Lick Observatory Eclipse Report. Whatever may be the law for a certain species of plates, the intensity will be a function of the corresponding exposure-time, or

$$\mathrm{I} = \phi\,(t)$$

If we could assume that the relative increase in the intensity of a square with the time was the same for all plates of different sensitiveness, then for any other species of plates the law is

$$\mathrm{I} = m\ (\phi)\ t$$

where m is the ratio of the intensities of any two standard-squares, exposed for the same number of seconds on the plates of the first and second species. As this is not as yet an established fact, it is advisable in all photometric work to confine one's self to one special species of plates. At the Lick Observatory the Seed-plates No. 26 are commonly used.

If we could assume that all the Seed-plates No. 26 were exactly alike, we might compare any square of any plate a of this species with any other square of any other plate b. But I have found that a square of a, exposed for t seconds, does not, in general, show the same intensity as a square of b, exposed for the same length of time. We may, however, confine ourselves to one single plate a, and then to another single plate b, and discover the law for each separately. And by taking a sufficient number of plates, and by combining the results suitably, we may come very near to the true relation of the time of exposure, and the consequent blackening of the film of this special species of plates.

Attacking the problem in this manner, I first took up some of the eclipse-plates of January 1st of this year.

Before going any further, it is necessary to say a few words about the method employed in comparing the standard-squares of a plate with each other. Let α and β be any two squares. To compare α and β all the remaining portions of the plate are covered with black cardboard. The plate, being fastened to a stand, is then set up, with the sky near the horizon as a background, and the BRASHEAR wheel photometer of the Lick Observatory (for a description of this instru-

ment, see the Lick Observatory Eclipse Report, pp. 84–85,) is placed between the eye and the brighter square of the two. The reading of the scale divided by one hundred gives the ratio of the photographic intensities of the two squares for that background. In order to avoid systematic errors in investigating the relative intensities of a series of squares, a square darker than the entire series should be selected as a comparison-square, and every square of the series should then be reduced to the comparison-square, by means of the photometer. Practically, only about four successive squares of our series lie within the range of the photometer. In order to compare the remaining darker squares of a series, we must invert the process and reduce the intensity of the comparison-square successively to that of every one of the remaining squares of the series. I have found that this process is subject to systematic errors. In order to avoid these errors, a standard-plate was specially exposed by Mr. BARNARD, according to a plan suggested by my previous trials. There are six groups, or sets, of standard-squares on this plate, each set containing four squares. Thus the first set contains exposures of 1, 2, 4, 8 seconds, the second exposures of 2, 4, 8, 16 seconds, the third of 4, 8, 16, 32 seconds, and so forth. The darkest square of each set (longest exposure) is the comparison-square for that set. It is easily seen that by eliminating the comparison-square from each set, any systematic errors arising from the change of the comparison-square and the choice of the background are avoided.

So far, the errors due to the difference in sensitiveness of various portions of the same plate had not been considered in the investigations. As a matter of fact, on all the plates previously observed there was only one square of one-second exposure, only one 2^s square, etc., and it did not suggest itself to me that the errors in these squares might be so large as to seriously affect the results. When, however, I took up the standard-plate just described, I at once saw that quite a difference exists between the intensities of the various squares of the same exposure-time, and it became necessary to take into account the probable errors of the intensities of the different squares. The following is the course taken in investigating this plate :

First, all squares of the same time of exposure,—as, for instance, all 8^s squares—were compared with a certain other square, the mean (denoted by the subscriptum ($_0$) as 8^s_0) taken, and the probable error of the mean determined. The difference in intensity of the different squares was surprising. Some squares were found to be from two to

three times as dark as others, exposed for the same length of time. To illustrate this, the comparisons of the 8^s squares are given here. In the following table $\frac{8_I}{32_{III}}$ = ratio of darkness of the 8^s square of the first set to that of the 32^s square of the third set. The numbers represent per cents., or, what is the same thing, the denominators are supposed to be equal to 100. The numbers in brackets represent the number of comparisons.

TABLE I.

Probable Error of an 8^s *Exposure.*

		%	
$\frac{8_I}{32_{III}}$	=	12.0 ± 0.3	(10)
$\frac{8_{II}}{32_{III}}$	=	8.4 ± 0.1	(5)
$\frac{8_{III}}{32_{III}}$	=	16.5 ± 0.2	(5)
$\frac{8_{IV}}{32_{III}}$	=	10.1 ± 0.3	(4)
$\frac{8_0}{32_{III}}$	=	11.8 ± 1.2	

From this we see that the 8_{III}^s square is twice as dark as the 8_{II}^s square, and we also find that even the mean of the four squares or the 8_0^s square has a propable error of 10%. In some cases this amounts to as much as 15%. The probable error of comparison might altogether be neglected in comparison with that of the square itself. This difference of intensity is due to changes in the sensitiveness of the film in different parts, as well as to changes in brightness of the standard flame. The nature of the background and the choice of the comparison-square cannot account for it, these being the same throughout the foregoing comparisons. Next, each of the six groups or sets of our standard-plate was taken up separately and compared with the comparison-square of that set. Each observation was then reduced to the corresponding mean square by multiplying by the ratio $\frac{\text{mean square}}{\text{square}}$. Thus, if in the first set we found $\frac{4_I^s}{8_I^s} = a$, then $\frac{4_0^s}{8_I^s} = a \times \frac{4_0^s}{4_I^s}$. We must remember that the numbers thus obtained do not necessarily express the absolute ratio of a certain square to the comparison-square, since a change of the comparison-square involves systematic errors. Hence, in each set the comparison-square was eliminated by taking ratios. It is for this reason that the results, so far obtained from

this plate, extend only to 32^s, since sets V and VI could not be compared by this method without introducing large errors of observation.

It seemed desirable to express the results obtained from sets I–IV absolutely in terms of some unit. An 8^s square occurred in each of the four sets, and after the 8^s square of each set had been reduced to the 8_0^s square every observation could be expressed in terms of the 8_0^s square—that is, in terms of the mean blackness of all the 8^s squares. Thus I supposed this square to contain 8 units, and then found the number of units contained in the other mean squares from each set. In doing this, I did not consider the single 1^s square of this plate, as it evidently was afflicted with a large error, being darker than the 2_0^s square. No systematic errors remain in these results, as the unit is independently determined for every set.

The following are the results of the investigations of this standard plate:

TABLE II.

Results from Standard Plate No. 1.

SET I.	SET II.	SET III.	SET IV.
Units.	Units.	Units.	Units.
$2_0^s = 2.1 \pm 0.3$	$2_0^s = [3.6 \pm 0.4]$	$4_0 = 4.7 \pm 0.6$	$8_0 = 8.0 \pm 0.0$
$4_0^s = 4.3 \pm 0.5$	$4_0^s = 3.8 \pm 0.2$	$8_0 = 8.0 \pm 0.0$	$16_0 = 10.7 \pm 2.2$
$8_0 = 8.0 \pm 0.0$	$8_0^s = 8.0 \pm 0.0$	$16_0 =$ (?)*	$32_0 = 14.6 \pm 2.6$

The law that the blackening of the film is proportional to the exposure-time is therefore confirmed within the limits of two seconds and eight seconds (within the limits of the accidental errors). On this plate the law no longer holds good after 8^s; the proportions fall off after that time. The exact nature of the curve could, of course, not be determined from the IVth set alone (on account of the large probable error remaining in the results).

After having determined the law for this special plate, I compared the observations previously made on other plates with the results just obtained, in order to test their validity. Only those observations were considered which could be freed from systematic errors. As before, in each series, the 8^s square was supposed to contain 8 units, and the other squares were then expressed in the same unit. It will be remembered that the results obtained from

* A wrong square was probably observed. The observation could not be repeated, on account of my absence from the Lick Observatory.

the first plate were gotten by using the mean of four squares, in almost all cases; but on the other plates we have only one square for every exposure-time. Hence (neglecting other circumstances), we must attribute to the results obtained from these other plates probable errors, which are at least twice as large as those found for the results of our standard-plate.

The results obtained from four plates are given in Table III. In one case, the value for the intensity-ratio $\frac{64^s}{128^s}$ has been added, it being impossible to express these squares in our unit without making additional comparisons; in another case the observed value for the relative intensities $\frac{16^s}{32^s}$, $\frac{32^s}{64^s}$, $\frac{64^s}{128^s}$, have been added. These ratios are free from systematic errors among themselves, but cannot as yet be combined with the remainder of the series without introducing them again. They contain, however, the errors arising from the errors of the squares themselves.

TABLE III.

Results from Four Plates.

	Standard Plate No. I.	Absorption Co-eff. Plate. June 10, 1889.	VOIGHT. B. Eclipse-Plate, Jan. 1, 1889.	VOIGHT. C. Eclipse-Plate, Jan. 1, 1889.	
	Units.	Units.	Units.		Units.
1^s	......	1.9 ±	......	3^s	2.5 ±
2^s	2.1 ± 0.3	2.5 ± 0.6	2.1 ± 0.6	4^s	3.2 ±
	4.3 ± 0.5				
4^s	3.8 ± 0.2	4.2 ± 0.8	4.0 ± 0.8	5^s	3.8 ±
	4.7 ± 0.6				
	8.0 ± 0.0			6^s	5.2 ±
8^s	8.0 ± 0.0	8.0 ± 0.0	8.0 ± 0.0		
	8.0 ± 0.0			7^s	6.9 ±
16^s	10.7 ± 2.2	19.2 ± 4.4	$\frac{16^s}{32^s} = \frac{24.0}{55.2}$	8^s	8.0 ±
32^s	14.6 ± 2.6	51.0 ± 5.2	$\frac{32^s}{64^s} = \frac{19.6}{24.0}$	9^s	8.1 ±
64^s	......	$\frac{64^s}{128^s} = \frac{13.2}{20.3}$	$\frac{64^s}{128^s} = \frac{8.4}{12.6}$	10^s	9.0 ±
128^s	......	......	......	11^s	9.5 ±

In every one of the four plates we found the law between 2^s and 8^s most strongly confirmed, and can now establish its validity for all the Seed-plates No. 26 (within the limits of the probable errors). Beyond 8^s the results obtained from our first plate are corroborated in the last plate only. In the second plate the proportionality goes as far as 16^s, and in the third even to 64^s, but in *no case does the proportionality go beyond* 64^s.

The results so far obtained are, in toto:

For the Seed-plates No. 26, the blackening of the film is proportional to the time of exposure within the limits of 2^s *and* 8^s, *and may be so as far as* 64^s, *but there is a* [strong] *probability that the proportions fall off after* 8^s.

In order to determine the exact position of the limit at which the proportionality begins to fall off, additional plates will be exposed according to a plan which will enable us to compare the darker squares, also, without introducing large errors. Between 2^s and 8^s, however, the evidence for the validity of the law is already sufficient.

BERKELEY, CAL., November, 1889.

THE LUNAR CRATER AND RILL—*HYGINUS.*

By EDWARD S. HOLDEN.

[ABSTRACT.]

I have asked Mr. BARNARD to make positive enlargements on glass of one of our best Moon negatives. A negative of August 14, 1888 (made by Mr. BURNHAM), has thus been enlarged two times, and shows the Moon, therefore, exactly as it would appear in the principal focus of a telescope 1140 inches, or 95 feet, long.* I find that I can use on this positive an eye-piece of one inch equivalent focus as a magnifier. That is, it is practicable to examine the lunar surface under perfect conditions of definition and illumination, and under a magnifying power of more than 1100 diameters, or, as if viewed by the naked eye, at a distance of 217 miles or so. This can be done whenever one pleases, and as long as one pleases.

As a test of the excellence of definition, I may mention a discovery which I have made on Mr. BARNARD's enlargement. It is well known that MAEDLER (and others) have mapped the walls of the

* The focus of our photographic lens is 570.2 inches.